A Modified Mass Concept could explain both the Dark Matter and the Dark Energy Phenomenon

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Abstract

Some consequences of a Modified Mass Concept (MMC) are discussed. According to MMC the inertial mass is not only determined by its energy, but also by a scalar field f depending on other masses. The concept consistently describes the galactic rotation curves, the inflation of the universe and its accelerated expansion, all without the necessity of Dark Matter (DM) and Dark Energy (DE). Instead, the effects attributed to DM are caused by a reduction of inertia acting as an enhancement of gravity. These results of MMC are similar to that of MOND. The effects usually attributed to DE in MMC stem from a new equation of state for baryonic matter, which always causes a negative pressure. In this respect the results are similar to those of the two component LCDM. In particular, according to MMC the late universe will pass into a state of constant energy density. Furthermore, the MMC can provide an explanation of the very high peculiar velocities found at large scales.

I Introduction

The "missing mass problem" at large scales is well-known in astronomy since Zwicky [1]. It applies to galaxies, clusters und the universe as a whole. In galaxies and clusters the observed mass cannot explain the observed dynamic. The amount of observed mass is always smaller than that of the calculated virial mass. The well-established solution for this problem is the postulation that there exists Dark Matter interacting with baryonic matter only by gravitation [2, 3, 4]. In the meantime, some concerns have been raised over the paradigm of DM [6, 7, 8, 9]. A popular alternative theory is the MOdified

Newton Dynamic (MOND) developed by M. Milgrom. Today there exists a lot of MONDian literature. A non-representative selection is e.g. [11, 12, 13, 14, 15, 16] and [17]. An extensive and actual list of MOND literature is in [18]. MOND is very successful with respect to the galactic rotation curves [19], galaxy formation [20] and the simulation of the evolution of spirals [21]. But in the past, there are also problems reported amongst others with lensing [22], with temperature profiles of cluster [23], with the Lyα forest [24] and the ominous "bullet cluster"[1].

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[1]an interesting discussion about this issue see [18]

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The "missing mass problem" of cosmology results from the observation that the density of the visible matter together with the DM is too small to cause the flatness of space, which is an established fact today. Furthermore, there is a need to explain why the universe had an accelerated expansion in the recent past (z= 0.5 . . . 1). This mostly is attributed to a special form of energy, called Dark Energy (DE). A good review on this topic is [25, 26]. According to the current conception, the universe today consists of about 70% DE and baryonic matter and DM add up to about 30%. The most simple variant of DE is the cosmological constant Λ [27]. The nowadays standard cosmology is the ΛCDM which combines the cosmological constant and the cold dark matter.

The presented work exhibits some parallels to both the MOND and the two component ΛCDM.

Section II deals with the basic idea of MMC. The issue of section III is the non relativistic equation of motion and the difference between MOND and MMC. Section IV is about the relativistic equation of motion in SR and GR. The new matter equation of state in MMC is deduced in section V. Finally, in section VI the predictions of MMC are compared to the observational facts.

II   The Basic Idea

In MMC the basic assumption is made that inertia of a mass m depends not only on its localized energy content, but there is also a non-local effect caused by a scalar field f, which another distant mass generates.

In the non-relativistic case of a test mass m at $\vec{r}$ in the gravitational field of a mass M causing the field f we may write:

$$m_{in} = f(\vec{r})m_g \qquad (1)$$

with the gravitational mass m_g and the inertial mass m_{in} of m with respect to M.

At first glance this seems to be the revival of a very out-dated debate. Didn't the famous Eötvös experiment [29, 30] and its successors [31, 32] and more recently [33] show that m_g is *equal to* m_{in} with very high precision, as we all have internalized? If we reconsider the exact conclusion of an Eötvös balance experiment, we realize that it has only shown the precise proportionality of the both quantities, and any multiplicative constant can be put into the definition of force. Thus, according to eq.(1) any Eötvös-like experiment fixed on earth will deliver a zero result in MMC too. However, MMC-effects could be detected in a satellite based experiment in which f changes[2].

According to eq.(1), f is dimensionless. The absolute strength of f will be not fixed here because it will be beneficial to scale f in such a way that $f \approx 1$ holds in the vicinity of a large mass. Then the absolute value of f can be merged with the gravitational constant G. Of course that would mean a rescaling of the gravitational strength.

Requirements on the scalar field f:

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[2]Unfortunately, the STEP mission [34], a test for the equivalence principle, will be flown in a near-circular orbit for different reasons. We may have to wait for LISA [35] the Laser Interferometer Space Antenna of NASA and ESA, designed for the detection of gravitational waves.

r1: $0 \leq f \leq 1$ because of scaling.

r2: $f \approx 1$ in the proximity of a large mass M.

r3: there exists a scale length λ depending on M.

r4: f decreases monotonically with the distance from M and at large distances r, $f \sim \lambda/r$ asymptotically holds.

The scale length λ is that distance from M at which the gravitational acceleration is a_C, thus $\lambda = \sqrt{\frac{GM}{a_C}}$. Therein is $a_C = \alpha c H_0$, c the speed of light, H_0 the Hubble constant of the present epoch and α is a fit parameter.[3,4]

In Special Relativity (SR), we have $m_g = m$ and $m_{in} = f m_{rel}$ with the rest mass m and $m_{rel} = \dfrac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$. In General Relativity (GR) the distinction between m_g and m_{in} is artificial, because the energy-momentum-tensor determines the metric. Thus, in GR we have de facto $m_g = m_{in} = f m_{rel}$.

Because mass plays a paramount role in all of physics, this basic assumption has momentous consequences. In this article I restrict myself to a few issues with direct impact on astronomy and cosmology. At the moment there is no field function f deduced from first principles. Therefore, in sections II to V consequences are discussed which do not depend on the special form of f. When comparing MMC with the observation in section VI empirical functions will be used.

[3] $\alpha = 0.169$ yields the MOND-Parameter a_M(see below)

[4] in the cosmological context λ depends on the asymptotic mass density (see below)

In this article the symbols have their usual meanings. For example, the momentum of a mass m with velocity $\vec{v}$ in MMC is

$$m_{in}\vec{v} = f\vec{p} \tag{2}$$

with $\vec{p} = m_{rel}\vec{v}$.

III   MMC and Newtonian Mechanics

A   Consequences of the Equation of Motion

The equation of motion of a test mass m in a Newtonian potential Φ_N of a large mass M is given by the second Newtonian law

$$\frac{d}{dt}\left(f(\vec{r}(t))m_g\vec{v}\right) = -m_g\nabla\Phi_N \tag{3}$$

or

$$f m_g\dot{\vec{v}} + \dot{f} m_g\vec{v} = -m_g\nabla\Phi_N \tag{4}$$

Applying d'Alembert's principle we realize that generally there are at least two inertial forces, even in a linear motion: the well-known inertial force $-f m_g\dot{\vec{v}}$ resisting a change of velocity and additionally $-\dot{f} m_g\vec{v}$ favoring the decrease of inertial mass, because this force is always repulsive. If m approaches M then $\dot{f} > 0$, whereas with increasing distance $\dot{f} < 0$ holds. In the cosmological context an analogue term will cause a negative pressure.

It is useful to solve eq.(4) for the acceleration

$$\dot{\vec{v}} = \frac{1}{f}(-\nabla\Phi_N - \dot{f}\vec{v}) \tag{5}$$

which gives the standard physics results for $f = 1$.

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One can see that the weak equivalence principle is valid, because the trajectory does not depend on m_g. Furthermore, we learn from eq.(3) that the momentum $f\vec{p}$ is conserved if $\nabla\Phi_N = \vec{0}$.

In the strict sense the strong equivalence principle holds only approximately if $f \approx const$, for example close to a large mass. Because the principle has to be valid only locally it is a good approximation especially in cosmology, where f changes only on very large scales.

Now let us discuss some consequences arising from the gravitational term.

Since, by assumption $f \leq 1$, the factor $1/f$ acts as an enhancement of gravity. In MMC a free falling body is accelerated by a factor $1/f$ more than in standard theory. This works like an additional gravitational mass. One can easily imagine that the enhancement factor could tremendously effect the development of structures at large scales. This applies to the formation of galactical spiral arms as well as to the emergence of voids and filaments at the level of super clusters.

In the end it accounts also for the observed galactical rotation curves. To show the flatness of the rotation curves in the outskirts of galaxies, we examine the special case of a test mass m moving in a center field of a large mass M. Below, m without a subscript always refers to the rest mass. The equations of motion in polar coordinates are

$$\ddot{r} - r\dot{\phi}^2 = \frac{1}{f}(-\frac{GM}{r^2} - f'\dot{r}^2) \qquad (6)$$

$$\frac{d}{dt}(mfr^2\dot{\phi}) = \frac{d}{dt}(fl) = 0 \qquad (7)$$

where $'$ means the derivative w.r.t. r and $fl = fmr^2\dot{\phi}$ is the magnitude of the angular momentum.

Because the scalar field f doesn't influence the direction of the gravitational force, the angular momentum vector stays fixed in space. Thus, eq.(7) expresses the conservation of angular momentum.

In a circular orbit, $\dot{r} = 0$ and $f' = 0$, and therefore the circular velocity is

$$v_C = r\dot{\phi} = \sqrt{\frac{1}{f}\frac{GM}{r}} \qquad (8)$$

At large distances ($r \gg \lambda$) we use $f \sim \lambda/r$ to obtain

$$v_C \sim \sqrt{\frac{GM}{\lambda}} = \sqrt[4]{GMa_C} = const \qquad (9)$$

showing that the circular velocity at large distances from the mass M becomes constant in MMC. MOND yields the same equation with $a_C = a_M$, where a_M is the MOND parameter[5]. This is what is observed at the outskirts of galaxies.

Unlike the DM paradigm the MMC requires not more gravitational, but less inertial mass.

Furthermore, there is a simple explanation for the fact that the virial mass M_{vir} is always larger than the observed mass M_{obs}.

The virial theorem can be derived by defining a quantity $\mathcal{G} = f\vec{p}\cdot\vec{r}$ [37]. The time average for a finite motion then is $\langle\mathcal{G}\rangle = 0$. The derivative of

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[5]in MONDian literature denoted as a_0 (see below)

$\mathcal{G}$ w.r.t. time and subsequent averaging together with eq.(3) yields $\langle 2fT \rangle = \langle \nabla U \cdot \vec{r} \rangle$, where T is the usually defined kinetic energy $T = \frac{1}{2}mv^2$. For the Newtonian gravitational potential, the resulting virial theorem in MMC is

$$2\langle fT \rangle = -\langle U \rangle \qquad (10)$$

Interestingly there is no expression resulting from the second inertial force. Thus, if $\langle fv^2 \rangle \approx \langle f \rangle \langle v^2 \rangle$ holds, the virial mass according to MMC is smaller than the Newtonian virial mass M_N by a factor $\langle f \rangle$

$$M_{MMC} \approx \langle f \rangle M_N \qquad (11)$$

We now discuss the asymptotic case $r \gg \lambda$ for the second term on the right-hand side of eq.(5), where we have $f \sim \frac{\lambda}{r}$.

Using $\dot{f} = \nabla f \cdot \vec{v} = f' \hat{r} \cdot \vec{v}$ this term, $-\dfrac{\dot{f}}{f}\vec{v}$, asymptotically becomes

$$-\frac{\dot{f}}{f}\vec{v} \sim -\frac{-\frac{\lambda}{r^2}\hat{r} \cdot \vec{v}}{\frac{\lambda}{r}}\vec{v} = \frac{\hat{r} \cdot \vec{v}}{r}\vec{v} \qquad (12)$$

which does not depend on λ! This means that at large distances the repulsion is independent of mass M.

Thus, the second term in eq.(5) represents nothing else but a non-relativistic analogue to the Hubble flux. A convincing argument in favor of this interpretation is the fact that in the cosmological context the position of each object may be written $\vec{r} = a(t)\vec{\xi}$ with the scale factor $a(t)$ and the constant comoving position $\vec{\xi}$. Then the term $-\dfrac{\dot{f}(|a\vec{\xi}|, \lambda)}{f(|a\vec{\xi}|, \lambda)}$ asymptotically becomes the Hubble function $H(t) = \dfrac{\dot{a}}{a}$. It depends neither on $\vec{\xi}$ nor on λ. However, a correct treatment of the Hubble flux is only possible within the framework of GR and will be discussed in sectionsV and VI.

B Energy Conservation

Now we turn to the energy conservation theorem for a test mass m whose potential energy U in the field of a large mass M is not explicitly time-dependent. The additional inertial force produces an additional term. Starting from eq.(3) and taking the scalar product with $\vec{v}$, one obtains

$$fm\dot{\vec{v}} \cdot \vec{v} + \dot{f}m\vec{v} \cdot \vec{v} = -\nabla U \cdot \vec{v} = -\frac{dU}{dt}$$

or

$$m\frac{d}{dt}(\frac{1}{2}f\vec{v}^2) + \frac{1}{2}\dot{f}mv^2 = -\frac{dU}{dt}$$

The total energy is now obtained by integrating over t

$$\frac{1}{2}fmv^2 + \frac{1}{2}\int \dot{f}mv^2 \, dt + U = E \qquad (13)$$

The meaning of the second term is discussed in sectionV.

C MMC vs. MOND

Today there is a completely relativistic version [28] of MOND. The difference between MOND and MMC however can be seen most clearly by analyzing the non-relativistic equation of motion. While in MMC eq.(3) holds, the corresponding equation in MOND according to [28] is

$$\mu\left(\frac{|\dot{\vec{v}}|}{a_M}\right)\dot{\vec{v}} = -\nabla\Phi_N \qquad (14)$$

where $\mu(x)$ is an empirical function with the following properties

$$\mu(x) = \begin{cases} 1 & x \to \infty \\ x & x \ll 1 \end{cases} \qquad (15)$$

and $a_M = 0.169cH_0 = 1.2\,10^{-10}m/s^2$ is the parameter of MOND.

The following function is often used for $\mu(x)$[19]

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \quad (16)$$

in the limit $|\dot{\vec{v}}| \ll a_M$ eq.(14) becomes

$$\frac{|\dot{\vec{v}}|}{a_M}\dot{\vec{v}} = -\nabla\Phi_N \quad (17)$$

As Bekenstein[28] pointed out, "MOND is characterized by a scale of acceleration a_M, not by a scale of length". This is a fundamental difference; MMC is characterized by a scale of length λ.

MOND has two interpretations [14]:

a) as a modification of the Newtonian equation of motion, as given by eq.(14), without a change of the gravitational field.

b) as a modification of gravity.
   In doing so a generalized Poison equation has to be solved

$$\nabla \cdot (\mu(\frac{|\nabla\tilde{\Phi}|}{a_M})\nabla\tilde{\Phi}) = 4\pi G\rho \quad (18)$$

   where ρ is the mass density. Together with the solution of eq.(18), the unchanged Newtonian equation of motion holds $\dot{\vec{v}} = -\nabla\tilde{\Phi}$.

In MMC, the gravitational potential and the structure of the equation of motion remain untouched, merely the definition of mass has changed.

IV MMC and Relativistic Mechanics

Below I use the notations $i, j, k, \ldots = 0, 1, 2, 3$ and $\alpha = 1, 2, 3$. The time coordinate has the subscript 0 and the signature is + - - -.

In order to obtain the law of energy conservation in SR, we first need a generalization of Newton's second law valid for all velocities. For a test mass m with the potential energy U which does not explicitly depend on time, the generalized second law is

$$\frac{d}{dt}\frac{fm\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = -\nabla U \quad (19)$$

Using the same procedure as in the derivation of eq.(13), it is easily found that

$$f\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - \int mc^2 \sqrt{1-\frac{v^2}{c^2}}\,\dot{f}\,dt + U = E \quad (20)$$

To obtain a covariant[6] equation of motion, I use the invariant four-dimensional element of length $ds = \sqrt{dx_i dx^i}$ and the dimensionless 4-velocity

$$u^i = \frac{dx^i}{ds} = (\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{\vec{v}/c}{\sqrt{1-\frac{v^2}{c^2}}})$$

and the resulting 4-momentum is

$$fp^i = fmcu^i$$

A Minkowski force can be defined with the 3-force $\vec{F} = -\nabla U$

$$K^i = (\frac{\vec{F} \cdot \vec{v}/c}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{\vec{F}}{\sqrt{1-\frac{v^2}{c^2}}})$$

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[6]in terms of invariance under Lorentz transformation

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The desired equation of motion using the proper time interval $d\tau = \sqrt{1 - \frac{v^2}{c^2}}\, dt$ is

$$\frac{d}{d\tau}(mcu^i) = \frac{1}{f}(K^i - mcw^i \frac{df}{d\tau}) \qquad (21)$$

with the 4-vector

$$w^i = (\frac{v^2/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{\vec{v}/c}{\sqrt{1 - \frac{v^2}{c^2}}})$$

and for u^i and w_i one obtains

$$u^i w_i = 0$$

Eq.(21) is the analog to eq.(5) and can be rewritten with $ds = cd\tau$

$$\frac{du^i}{ds} = \frac{1}{f}(\frac{K^i}{mc^2} - w^i \frac{df}{ds}) \qquad (22)$$

the time coordinate of eq.(22) yields the energy conservation eq.(20), the space coordinates of eq.(22) the three dimensional equation of motion (19).

In GR gravitation is included in the metric g^{ik}. Hence, we set $K^i = 0$ in eq.(22) and generalize to arbitrary coordinates by using the covariant derivative instead of the partial one. Then we find the desired result for GR

$$\frac{du^i}{ds} + \Gamma^i_{kl} u^k u^l = -\frac{1}{f}\frac{df}{ds} w^i \qquad (23)$$

Now u^i and w^i generally depend on g^{kl}. Γ^i_{kl} are the Christoffel symbols.

V   MMC and Cosmology

A   The Equation of State

In this section a is the scale factor with dimension of length.

Now I discuss the physics of the second inertial force and its justification. An adequate model for homogeneous and isotropic matter in cosmology is that of a perfect fluid, whose energy-momentum-tensor is given by

$$T^{ik} = (\epsilon + P)u^i u^k - \rho g^{ik} \qquad (24)$$

P is the pressure, ϵ the energy density in the proper system.
It describes the background and therefore we have to replace f by f_b in this context.
If we choose a synchronous reference system, then $x^0 = c\tau$ as well as $g_{00} = 1$ and $g_{0\alpha} = 0$. Otherwise g_{00} and u^0 would depend on the gravitational potential Φ. Then

$$T^{00} = \frac{\epsilon + P}{\sqrt{1 - \frac{v^2}{c^2}}^2} - P = \frac{\epsilon + P\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \qquad (25)$$

Since T^{00} is the energy density in the frame of reference, it is determined by the energy conservation law (20). For that purpose we set $U = 0$ and obtain

$$f_b \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - \int_0^\tau mc^2 \sqrt{1 - \frac{v^2}{c^2}} \frac{df_b}{d\tau'}\, d\tau' = E \quad (26)$$

Now in eq.(26) we switch over to densities remembering that the volume has to be transformed too.

$$f_b \frac{\rho c^2}{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \int_0^\tau \rho c^2 \sqrt{1 - \frac{v^2}{c^2}} \frac{df_b}{d\tau'}\, d\tau' = \frac{E}{V}$$

setting

$$F(\tau) = \int \sqrt{1 - \frac{v^2}{c^2}} \frac{df_b}{d\tau}\, d\tau$$

we obtain

$$\rho c^2 \frac{f_b - \frac{F(\tau) - F(0)}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v^2}{c^2}\frac{F(\tau) - F(0)}{\sqrt{1 - \frac{v^2}{c^2}}}}{1 - \frac{v^2}{c^2}} = \frac{E}{V}$$

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Now we compare this with eq.(25) keeping in mind that there ϵ and P are quantities in the proper system, where $F(\tau) = f_b(\tau)$. If we refer to cosmic time, then $f_b(0) = 1$ is valid, because the initial state is characterized by an extreme density of mass and energy respectively.

Thus,

$$\epsilon = \rho c^2 \qquad (27)$$

$$P = (f_b - 1)\rho c^2 \qquad (28)$$

Eq.(28) is the equation of state of matter in MMC. According to MMC matter always shows negative pressure except from $f_b = 1$.

B   Cosmological Consequences

In this subsection the subscript 0 denotes the present epoch.
Below, radiation will be neglected and ΛCDM always refers to the two component ΛCDM.

In the cosmological context the scalar field f_b is considered as a function of the scale factor a. Often the dimensionless state of equation w is used [26]

$$w = w(a) = \frac{P}{\epsilon} = f_b - 1 \qquad (29)$$

While in MMC w is a function of the scale factor a, w is constant in ΛCDM. For pressureless matter ΛCDM yields $w_m = 0$, and for the cosmological constant Λ $w_\Lambda = -1$ holds. In other scalar field models of DE, e.g., the Quintessence [25] $w = w(\varphi)$ with a scalar field φ holds.

The basic equations of cosmology are:
the two Friedmann equations (here without Λ)

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \qquad (30)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) \qquad (31)$$

as well as the continuity equation

$$\dot{\rho} + 3H(\rho + \frac{P}{c^2}) = 0 \qquad (32)$$

and the equation of state(28).
We substitute eq.(28) into eq.(31) and obtain

$$\frac{\ddot{a}}{a} = -4\pi G\rho(f_b - \frac{2}{3}) \qquad (33)$$

From this it follows the important result that for $f_b < 2/3$ the expansion of the universe is accelerated ($\ddot{a} > 0$). This is what we observe.

Substituting eq.(28) into eq.(32) we find

$$\dot{\rho} + 3H f_b \rho = 0 \qquad (34)$$

From this continuity equation we obtain also some interesting results.

The solution of eq.(34) is

$$\rho(a) = \rho_0 e^{-3\int_{a_0}^{a} \frac{f_b(a')}{a'} da'} \qquad (35)$$

The scalar field may be written as $f_b(a) = 1 - \phi(a)$[7]. Thus,

$$\rho(a) = \rho_0 (\frac{a_0}{a})^3 e^{3\int_{a_0}^{a} \frac{\phi(a')}{a'} da'} \qquad (36)$$

and defining the enhancement factor

$$\Gamma(a) = e^{3\int_{0}^{a} \frac{\phi(a')}{a'} da'} \qquad (37)$$

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[7]ϕ is quasi a measure for the deviation from standard theory

we obtain

$$\rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^3 \frac{\Gamma(a)}{\Gamma(a_0)} \tag{38}$$

The attenuation effect of the pressureless matter $\propto 1/a^3$ during the expansion is separated as a factor. Since $f_b(a)$ asymptotically approaches zero, $\phi(a)$ trends towards 1. Therefore, according to eq.(36), ρ approaches a positive limit ρ_∞. Thus, the universe finally trends to a constant energy density.

To distinguish analog quantities in MMC from their counterpart in ΛCDM the former will be denoted with $\tilde{\ }$.
It is intuitively obvious that we can define a cosmological constant in MMC in this way:

$$\rho_\infty = \tilde{\rho}_\Lambda = \frac{c^2}{8\pi G} \tilde{\Lambda} \tag{39}$$

Thus, the cosmological scale length λ is also found

$$\lambda = \frac{1}{\sqrt{\tilde{\Lambda}}} = \frac{\sqrt{3}c}{H_\infty} \tag{40}$$

In the cosmological context λ depends on the asymptotic density of mass and hence on the asymptotic Hubble constant H_∞.

Now we will divide the density ρ in the two fictitious (!) parts $\tilde{\rho}_m$ and $\tilde{\rho}_{DE}$, which in ΛCDM correspond to the pressureless (baryonic as well as Dark) matter and the DE in terms of the cosmological constant. But, in MMC $\tilde{\rho}_{DE}$ isn't constant.

Once again we have to keep in mind that in MMC there is neither DM nor DE!

With

$$\rho = \tilde{\rho}_m + \tilde{\rho}_{DE} \tag{41}$$

and

$$\tilde{\rho}_m = \tilde{\rho}_{m,0} \left(\frac{a_0}{a}\right)^3 \tag{42}$$

we find

$$\tilde{\rho}_{DE} = \rho_0 \left(\frac{a_0}{a}\right)^3 \left(\frac{\Gamma(a)}{\Gamma(a_0)} - \frac{\tilde{\rho}_{m,0}}{\rho_0} \right) \tag{43}$$

Since pressure as well as $\tilde{\rho}_{DE}$ vanish if $a \ll \lambda$ and using $\Gamma(0) = 1$ we find

$$\Gamma(a_0) = \frac{\rho_0}{\tilde{\rho}_{m,0}} \tag{44}$$

Thus, we can rewrite eq.(38) as

$$\rho = \tilde{\rho}_m \, \Gamma(a) \tag{45}$$

Assuming that ρ is always critical and with the scaled density eq.(45) becomes

$$\tilde{\Omega}_m \, \Gamma(a) = 1 \tag{46}$$

or,

$$\tilde{\Omega}_{DE} = \tilde{\Omega}_m(\Gamma(a) - 1) \tag{47}$$

Since eq.(34) has constant solutions for $f_b = 0$, a simple possibility opens up to explain the occurrence of the inflation.
Let us assume that at Planckian time the scalar field f_b has the value zero and the inflation began at t^*. If the value of the scalar field switches to 1 at a later time t_i by symmetry breaking, then this results in a very high constant density ρ^* during $t^* \geq t \geq t_i$. According to eq.(28) a very high negative pressure occurs that fuels the inflation. At time t_i the inflation stops abruptly.

So in MMC the initial and the final state of the universe is characterized by constant densities and thus by massless matter (de Sitter universe).

The well-known de Sitter solution during inflation is

$$a(t) = a^* e^{H^*(t-t^*)} \quad \text{with} \quad H^* = \sqrt{\frac{8\pi G}{3}\rho^*}$$

$$\tag{48}$$

and in the final state

$$a(t) = a_0 e^{H_\infty(t-t_0)} \quad \text{with} \quad H_\infty = \sqrt{\frac{8\pi G}{3}\rho_\infty}$$

$$\tag{49}$$

The flatness condition of space ($k = 0$) follows from of the Friedmann eq.(30)

$$k = \frac{a^2}{c^2}(\frac{8\pi G}{3}\rho - H^2)$$

$$= \frac{8\pi G}{3c^2}\rho_0\frac{a_0^3}{\Gamma(a_0)}\frac{\Gamma(a)}{a} - \frac{\dot{a}^2}{c^2} = 0 \tag{50}$$

using

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_0\frac{a_0^3}{\Gamma(a_0)}\frac{\Gamma(a)}{a} = H_0^2\frac{a_0^3}{\Gamma(a_0)}\frac{\Gamma(a)}{a}$$

we obtain

$$\int_0^a \frac{\sqrt{a'}\,da'}{\Gamma(a')^{1/2}} = H_0\frac{a_0^{3/2}}{\Gamma(a_0)^{1/2}}t \tag{51}$$

In section VI it is shown that eq.(51) delivers the correct age of the universe.

VI   MMC and Observations

Since up to now there is no field function f derived from first principles, two empirical functions will be tested for their ability to reproduce various observations. In the cosmological context as background field function, I used

$$f_1(x) = 1 - e^{-1/x} \qquad \text{with} \qquad x = \frac{a}{\lambda}$$

For the galactical rotation curves, the function

$$f_2(x) = \frac{1}{\sqrt{1+x^2}} \quad \text{with} \quad x = \frac{r}{\lambda}$$

is used. The function f_2 is related to the empirical function of MOND (eq.(16)) by $f_2(x) = \mu(1/x)$.

A   Galactical Rotation Curves

By means of two arbitrarily selected examples, the NGC 5033 and the low surface-brightness galaxy UGC 128, it is shown that MOND and MMC yield very similar results. The measurements used are from [19].

The method chosen here is geared to the approach of MOND in [19]. There, at first the enclosed total mass $M_t(r)$ of a circular orbit with radius r is divided into the disk mass M_d, the gas mass M_g and bulge mass M_b. Thus $M_t = M_d + M_g + M_b$. The examples are chosen so that $M_b = 0$ to minimize the uncertainties. The partitioning was made under the assumption that Newtonian mechanics is valid. Hence,

$$M_t(r) = \frac{(v_d^2 + v_g^2)r}{G}$$

Now the MOND eq.(14) becomes

$$\mu\left(\frac{\dot{v}}{a_M}\right)\dot{v} = \frac{GM_t(r)}{r^2} \tag{52}$$

It has to be solved for the acceleration $\dot{v}$. Then the rotation curve velocity is given by

$$v_{MOND}(r) = \sqrt{r\dot{v}(r)} \tag{53}$$

In MMC instead of eq.(52) one has to solve the following eq.

$$\frac{1}{f(r)}\frac{GM_t(r)}{r^2} = \frac{v^2}{r} \tag{54}$$

So we find

$$v_{MMC} = \sqrt{\frac{1}{f(r)}(v_d^2 + v_g^2)} \qquad (55)$$

The scale length λ is given by

$$\lambda(r) = \sqrt{\frac{GM_t(r)}{a_C}}$$

with the fit parameter α defined by $a_C = \alpha c H_0$.

The rotation curves fig.1 and fig.2 are computed with the empirical function f_2 also used in MOND. The results are very insensitive to the chosen function. The function f_1 delivers almost the same results with a somewhat different fit parameter. For f_1 we find $\alpha = 0.101$ and for f_2 $\alpha = 0.135$; for comparison the MOND fit was performed with $a_M = 0.169 c H_0$.

Both examples suggest that whenever MOND works MMC will work too.

B Cosmological Aspects

The cosmological MMC model needs 3 input parameters: $\tilde{\Omega}_{m,0}$, the normalized density of the pressureless matter of the present epoch, H_0 the present Hubble constant and k, the curvature parameter of the metric.

The values used here are: $\tilde{\Omega}_{m,0} = 0.25$, $H_0 = 0.237 \cdot 10^{-17}/s$ and $k = 0$.

The flatness of space is presumed not explained. MMC is a "one component" model. However, because space is flat this single component must be critical.

Hence a series of predictions arise agreeing more or less with the observations.

The following computations are performed with function f_1. Analogous expressions for f_2 are

easily found. The results are not strongly dependent on the chosen function.

It should be remembered that the functions used are merely empirical. Therefore, we should not expect a perfect correlation with observation.

From $f_1 = 1 - e^{-\lambda/a}$ we find the enhancement factor[8]

$$\Gamma(a) = e^{3Ei(1,\lambda/a)} \qquad (56)$$

We can rewrite eq.(46) for the present epoch

$$\tilde{\Omega}_{m,0}\, e^{3\,Ei(1,\lambda/a_0)} = 1 \qquad (57)$$

Immediately there result some numerical values: $\Gamma(a_0) = 4.00$, the ratio $x_0 = a_0/\lambda = 1.690$ and $f_1(x_0) = 0.447$.

Substituting $f_b = f_1$ in eq.(38) we find for $a \to \infty$

$$\rho_\infty = \rho_0 \left(\frac{a_0 e^{-\gamma}}{\lambda e^{Ei(1,\lambda/a_0)}}\right)^3 \qquad (58)$$

where $\gamma = 0.5772\ldots$ is the Eulerian constant.

Therefore, $\rho_\infty = 0.214\rho_0$ and $H_\infty = 0.462 H_0$. Setting $\rho_{c,0} = 1.01 \cdot 10^{-26}\ kg/m^3$ we find with eq.(39) and eq.(40)
$\tilde{\Lambda} = 0.403 \cdot 10^{-52}\ m^{-2}$, $\lambda = 0.158 \cdot 10^{27} m$ and $a_0 = 0.266 \cdot 10^{27} m$.

Comparing with ΛCDM ($\Omega_{\Lambda,0} = 0.7$), it is clear that the cosmological constant $\Lambda = 1.32 \cdot 10^{-52}\ m^{-2}$ is slightly larger than $\tilde{\Lambda}$.

Substitution of the values above into eq.(51) delivers the age of the universe as $t_0 = 12.9 Gy$,

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[8]For the function $Ei(1,z)$ see [38], there denoted as $E_1(z)$.

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which has the right order of magnitude.

The two component ΛCDM and MMC mostly produce similar results. This shall be shown with the help of 3 examples.

α) The normalized density of the cosmological constant in ΛCDM is (see [25])

$$\Omega_\Lambda = \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}(\frac{a_0}{a})^3 + \Omega_{\Lambda,0}} \tag{59}$$

With eq.(47) MMC yields

$$\tilde{\Omega}_{DE} = \tilde{\Omega}_{m,0}(\frac{a_0}{a})^3(e^{3Ei(1,\lambda/a)} - 1) \tag{60}$$

Both functions are shown in fig.3. The curves cross over at x_0 if $\tilde{\Omega}_{m,0} = \Omega_{m,0}$.

β) The luminosity distance d_L in ΛCDM with the redshift parameter z

$$\frac{a_0}{a} = 1 + z \tag{61}$$

is according to [25]

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{\Lambda,0}}} \tag{62}$$

In MMC we find with eq.(57)

$$d_L = \tag{63}$$
$$\frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{(1+z')^3 e^{3(Ei(1,\frac{1+z'}{x_0})-Ei(1,\frac{1}{x_0}))}}}$$

For a comparison, see fig.4

γ) In ΛCDM the accelerated expansion of the universe began at [25]

$$z_c = (\frac{2\Omega_{\Lambda,0}}{\Omega_{m,0}})^{1/3} - 1$$

With $\Omega_{\Lambda,0} = 0.7$, this gives $z_c = 0.67$. Substituting $\tilde{\Omega}_{\Lambda,0} = 0.75$ we find $z_c = 0.82$.
In MMC we have to solve $f_1(x_{2/3}) = 2/3$ and find $x_{2/3} = 0.910$. This together with eq.(61) gives $z_{2/3} = 0.86$

A substantial difference between standard physics and MMC is that according to the former the peculiar motion of a body in a complete homogeneous and isotropic expanding universe will finally come to rest in the Hubble flux, while in MMC this is not the case.

In standard physics, a body with momentum p w.r.t. a comoving system (see textbooks, e.g. [36])

$$p\,a = \frac{mva}{\sqrt{1 - \frac{v^2}{c^2}}} = const$$

hence the velocity v decreases while a increases. In MMC in contrast, the momentum is fp, thus

$$fp\,a = \frac{fmva}{\sqrt{1 - \frac{v^2}{c^2}}} = const$$

If $f \propto \lambda/a$ holds then v is constant. This could help to understand an observation reported by Watkins et al. [10]. They found that there is an exceptional large bulk flow[9] of 407 km/s within a Gaussian window of radius $50h^{-1}$ Mpc. According to the ΛCDM cosmology the value should be about 100 km/s, which is significantly too small. Certainly inhomogeneities will have a large influence on this phenomenon. However, MMC could at least partly explain the observed large value.

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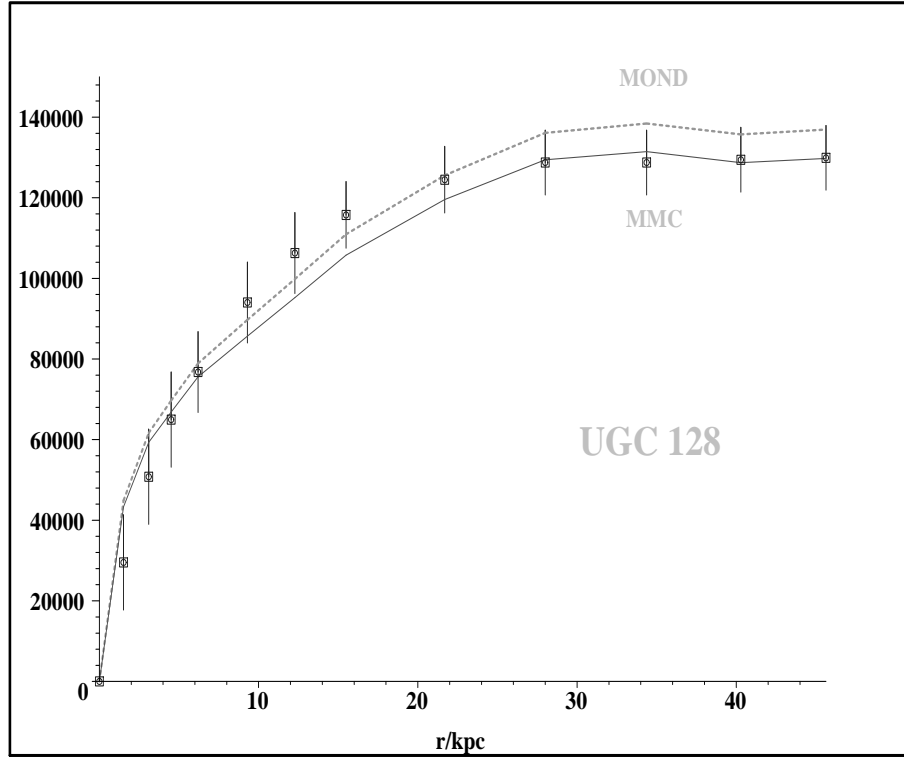[9]dipole moment of the peculiar velocity field

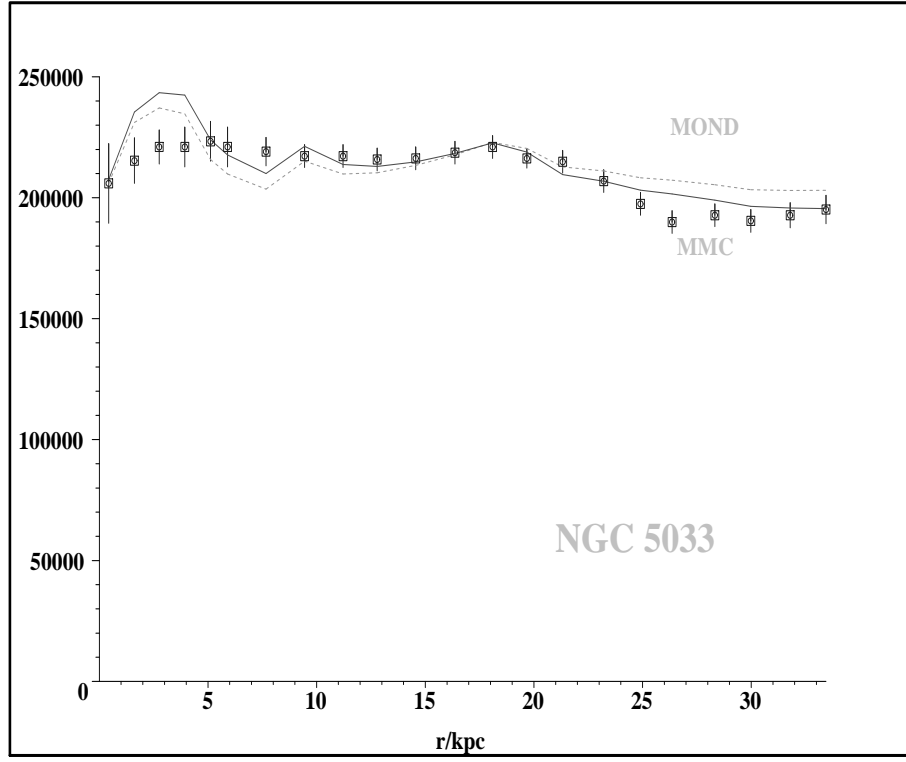Figure 1: Rotation curve of UGC 128. Velocity in m/s. MMC solid line; MOND dashdot line. Data points from [19]

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Figure 2: Rotation curve of NGC 5033. Velocity in m/s. MMC solid line; MOND dashdot line. Data points from [19]
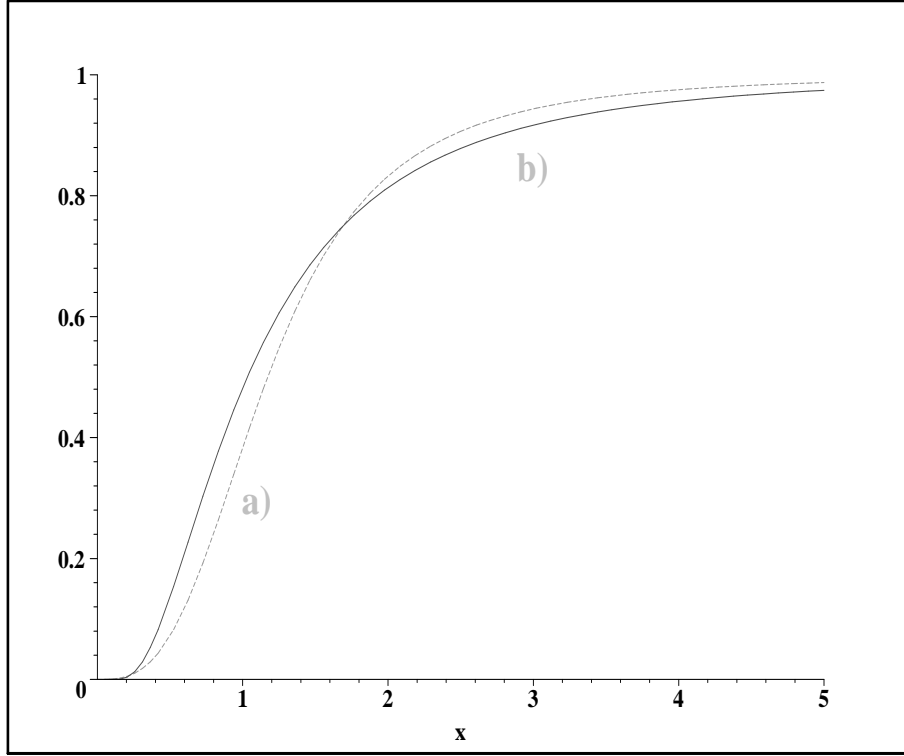
Figure 3: a) Ω_Λ and b) $\tilde{\Omega}_{DE}$ as a function of $x = a/\lambda$
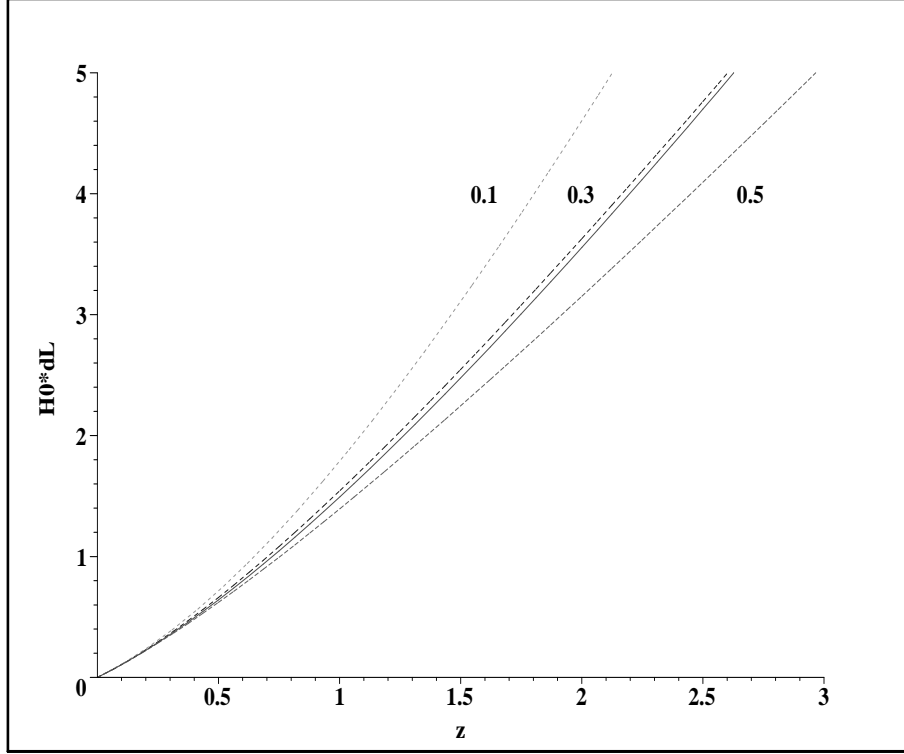
Figure 4: $H_0 \cdot d_L$ vs. z. Solid line is MMC. The parameters are $\Omega_{m,0}$ for ΛCDM

VII   Summary

Some consequences of the assumption that the inertial mass depends not only on its rest energy but also on a scalar field which is caused by other mass are discussed. Applying this assumption to the equation of motion we find that besides the well-known inertial force, $-m\vec{v}$, a further inertial force emerges. This force is always repulsive and favors the decrease of inertial mass. The acceleration caused by this force becomes independent of mass at large distances. Thus, it is related to the Hubble flux.

A reduction of inertia on the other hand acts as an enhancement of gravity as if there would be more gravitational mass. This effect explains the galactical rotation curves. The assumption of a DM isn't necessary at all. Unlike the DM paradigm the MMC postulates not more gravitational but less inertial mass.

It is easy to imagine that the enhanced gravity will strongly effect the formation of structures at large length scales. It effects the formation of spiral arms of galaxies just as well as the development of voids and filaments at the level of super clusters.

A new equation of state for baryonic matter has some intriguing properties. For instance, baryonic matter is always accompanied by a negative pressure. Therewith, it is a consequence of the Friedmann equation that for $f_b < 2/3$ the expansion of the universe is accelerated. Moreover, the continuity equation allows constant solutions for $f_b = 0$. Thus, the late universe approaches a state of constant energy density. In this regard, the MMC is similar to ΛCDM. It also opens up the possibility to understand the inflation as a phase transition with f_b as the order parameter. We assume that immediately after Planckian time the scalar field has the value zero. During this period inflation occurs. Later on when the value of f_b switches to 1 by symmetry breaking the inflation stops. Thus, the initial and the final state of the universe can be understood in a very similar way: it is a state without inertia.

Using the empirical functions the computed age of the universe, the luminosity distance and the redshift z of the beginning of the accelerated expansion comply to large extent with the values of the two component ΛCDM with $\Omega_\Lambda = 0.7 \ldots 0.75$. Therefore, also DE isn't necessary as explanation. Furthermore, the MMC can provide an explanation for the very high peculiar velocities found at large scales.

From the fact that MMC requires neither DM nor DE to explain some phenomenons it is incorrect to conclude that these don't exist. Maybe they do. If MMC is correct, then their energy density should be much smaller as assumed now.

MMC is just an idea, not yet a theory. A lot of questions are still open. The modest aim of this article was to demonstrate the potential of the idea.

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